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Threebrane absorption and emission from a brane-antibrane system

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Abstract: We show that a previously proposed model based on a D3-brane–anti-D3-brane system at finite temperature can reproduce the low-frequency absorption and emission probabilities of the black threebrane of Type IIB supergravity arbitrarily far from extremality, for arbitrary partial waves of a minimal scalar field. Our calculations cover in particular the case of the neutral threebrane, which corresponds to the Schwarzschild black hole in seven dimensions. Our results provide not only significant evidence in favor of the brane-antibrane model, but also a rationale for the condition that the energies of the two component gases agree with one another. In the course of our analysis we correct previous results on the absorption probabilities of the near-extremal threebrane, and extend them to the far-from-extremal regime.

Keywords: D-branes, Black Holes in String Theory, Tachyon Condensation, AdS-CFT and dS-CFT Correspondence.

Contents

1. Introduction

Recent work has opened the possibility to attain a quantitative understanding of the physics of black holes *far* from extremality, an important and long-standing problem. In particular, three years ago a study of brane-antibrane systems at finite temperature led the authors of [1, 2] to construct a microscopic model for the black threebrane of Type IIB supergravity and the black twobrane and fivebrane of eleven-dimensional supergravity. The model is based on decoupled stacks of branes and antibranes, with a gas of massless particles on each stack, and was shown to successfully reproduce the corresponding entropies arbitrarily far from extremality.[1] It also correctly accounts for various other properties of the black branes; in particular, their negative specific heat and pressure find a natural explanation in terms of brane-antibrane annihilation.

In the past few months, these results have been generalized in various directions. It has been shown that the brane-antibrane model predicts the correct entropy not only for the three non-dilatonic cases studied in [1], but also for other singly- and multiply-charged black branes [4]–[9] (see also the older works [10, 11]). The model has been found to reproduce even the highly non-trivial entropy formulas for branes rotating with arbitrary amounts of angular momentum [3, 4]. Moreover, preliminary numerical calculations in the

[1] For a list of other approaches to the far-from-extremal problem, see, e.g., [3].

relevant strongly-coupled gauge theory [4] (using the methods developed in [12]) appear to support the result of [1] regarding the stability of the brane-antibrane system at high enough temperatures, which is in turn one of the key assumptions of the model.

In spite of this already quite significant body of evidence, there are at least two aspects of the brane-antibrane model that are still not fully understood. One is the fact that the model accounts for the exact functional form of the entropy, but it yields a numerical coefficient that, at least under the assumptions adopted in [1], is a power of two too small. Intriguingly, in all (singly-charged) cases the discrepancy can be summarized by the succinct but evidently unphysical statement that the supergravity entropy behaves as if each of the gases in the model had access to twice as much energy as is available to it.[2] What is needed then is a physically plausible reinterpretation of this seemingly simple pattern.

In [4] it was noted that it is best to trace the discrepancy back to a disagreement between the supergravity and microscopic *masses* (rather than entropies), which would in turn imply that some type of binding energy has not been properly taken into account in the microscopic side. This point of view is physically sensible, and we will have more to say about it and a closely related possibility in the Conclusions. Nevertheless, given the simple form of the discrepancy one would hope to be able to pin down its origin quantitatively.

In [6, 8] a different, 'empirical' interpretation of the discrepancy was proposed, in terms of the allocation of the total gas energy to only *one* of the gases (or the corresponding fraction of *each*), and an accompanying reduction by a factor of four in the value of the component brane tension. Again, to be satisfied one would like to derive both of these conditions dynamically. For instance, one could try to associate the reduction in the tension with *partial* brane-antibrane annihilation, which could then also conceivably be responsible for the reduction in the number of gas degrees of freedom. But such an interpretation would run into trouble for the charged black brane, because in that case the number of branes and antibranes is not equal, and the surplus branes (or antibranes) would not be able to annihilate. Moreover, as already mentioned, the results of [1, 4] appear to indicate that the brane-antibrane system is in fact stable, and therefore no such partial annihilation should take place. It seems then that one must regard the proposed reduction in brane tension as originating from binding energy, which essentially brings us back to the interpretation of [4], except that here we are in addition missing an explanation for the putative 'deactivation' of all but one of the gases.

A second (and possibly more significant) puzzling feature of the model is the fact that, to correctly reproduce the entropy of black branes with non-zero charge, the gases involved must be assumed to have equal energies, and consequently different temperatures. Since the gases are assumed to be decoupled from one another, their having different temperatures is not entirely out of the question, but up to now we do not understand why it is that their energies (or equivalently, their pressures) must agree.[3] In other words, unless there is some

[2] As shown in [6], for multiply-charged systems the energy carried by each gas would have to be a factor $2^{1+(K-1)/\lambda}$ larger than is physically possible, where K denotes the number of charges and λ specifies the energy dependence of the gas entropy, $S \propto E^{\lambda}$.

[3] In the interpretation of [6, 8], the condition that the gas energies be the same is replaced by the equally

physical restriction, in the microscopic side one could construct a family of systems which differ only in the way the total gas energy is split among the component gases, and each of these systems would be expected to have a supergravity counterpart. Families of *regular* supergravity solutions with the desired properties are not known to exist, so in [1] it was speculated that perhaps the condition that the gas energies be equal in the microscopic side could somehow correspond to the absence of singularities in the supergravity side.[4]

Clearly the brane-antibrane model will not be on firm ground until these two aspects are properly understood. In the meantime, and as part of the strategy to achieve that goal, it is important to subject the model to further tests, and in particular to compute quantities other than the entropy. Initial steps in this direction were taken already in [1], where it was shown that the model implies the correct form for the supergravity energy-momentum tensor, and a transverse size (due to thermal fluctuations) for the microscopic system which is of the same order as the horizon radius. (This last property has also been seen in [4, 5, 9], and is in consonance with the proposal of [13].) In that same work, a study of the thermodynamic counterpart [14] of the Gregory-Laflamme instability [15] led to the satisfying conclusion that the microscopic system, while unstable to collapse when wrapped on a sufficiently large torus, would actually stabilize at a finite size and acquire properties coinciding with those of a ten- or eleven-dimensional black *hole*. Very recently, the thermalization rate of the brane-antibrane system was studied in [16], and compared against the quasi-particle picture developed in [17] (see also [18]), again with satisfying results.

In this paper we will take additional steps in this direction, by studying the manner in which the microscopic system absorbs and emits radiation. More specifically, we will compare the absorption probabilities and Hawking radiation rates predicted by the brane-antibrane model against the actual supergravity results, to lowest order in the radiation frequency. For concreteness, we will restrict our analysis to radiation associated with a minimal scalar field, in the presence of a black threebrane with arbitrary charge. Given the successful generalizations in [3]–[9], we would expect analogous results for other types of black brane.

We start in section 2 by reviewing the results of the brane-antibrane model for the threebrane case. We next work out in section 3 the absorption probabilities for the microscopic system. This requires an analysis of the corresponding probabilities in the case of a *near-extremal* threebrane, which are determined in section 3.1, correcting previous results. The absorption probabilities predicted by the model are then explicitly written down in section 3.2, and shown to agree with their supergravity counterparts in section 4, first for the previously examined neutral case in section 4.1, and then for the newly computed charged case in section 4.2. Finally, in section 5 we carry out a successful comparison between the rates of Hawking radiation in the microscopic and supergravity sides. We conclude in section 6, which includes both a summary of our results in section 6.1 and a critical discussion on the assumptions of the model in section 6.2.

puzzling condition that only one of the gases contributes.

[4]This possibility has been raised independently by David Lowe.

2. Review of the brane-antibrane model

The metric of the black threebrane solution of Type IIB supergravity takes the form

$$ds^2 = \frac{1}{\sqrt{H(r)}}\left(-f(r)dt^2 + d\vec{x}^{\,2}\right) + \sqrt{H(r)}\left(\frac{dr^2}{f(r)} + r^2 d\Omega_5^2\right),$$

where

$$H(r) = 1 + \frac{r_h^4 \sinh^2\alpha}{r^4}, \qquad f(r) = 1 - \frac{r_h^4}{r^4},$$

with r_h the horizon radius. The ADM mass density of this geometry is

$$m_{\rm SG} \equiv \frac{M_{\rm SG}}{V} = \frac{\pi^3}{\kappa^2} r_h^4 \left(\frac{3}{2} + \cosh 2\alpha\right), \tag{2.1}$$

its entropy density

$$s_{\rm SG} \equiv \frac{A_h/4G_N}{V} = \frac{2\pi^2}{\kappa^2} r_h^5 \cosh\alpha, \tag{2.2}$$

and its Hawking temperature

$$T_H = \frac{1}{\pi r_h \cosh\alpha}. \tag{2.3}$$

The solution also involves a RR five-form field-strength, associated with a charge

$$Q_{\rm SG} = \frac{\pi^{5/2}}{\kappa} r_h^4 \sinh 2\alpha. \tag{2.4}$$

It was shown in [1] that $s_{\rm SG}(m_{\rm SG}, Q_{\rm SG})$ can be reproduced with a field-theoretic model based on a system of N D3-branes, $\bar{N}$ anti-D3-branes, and two gases of $\mathcal{N}=4$ super-Yang-Mills (SYM) particles, arising respectively from the massless modes of 3-3 and $\overline{3}$-$\overline{3}$ open strings. The stack of branes and that of antibranes (together with their corresponding gases) are assumed not to interact with one another. The charge of the microscopic system is then

$$Q_{\rm FT} = N - \bar{N}, \tag{2.5}$$

and its total mass density,

$$m_{\rm FT} = (N + \bar{N})\tau_3 + e + \bar{e}, \tag{2.6}$$

with $\tau_3 = \sqrt{\pi}/\kappa$ the D3-brane tension and e ($\bar{e}$) the energy density of the gas on the D3-branes ($\overline{\rm D3}$-branes). As mentioned in the Introduction, the model assumes that $e = \bar{e}$.

The entropy of the system is entirely due to the two $\mathcal{N}=4$ SYM gases. In the regime of interest, $g_s(N-\bar{N}) \gg 1$ (where the supergravity solution is reliable), SYM is strongly-coupled, and so the entropy of the two gases cannot be determined perturbatively. It is however known [19, 20] via the AdS/CFT correspondence [21],

$$s_{\rm FT} = 2^{5/4} 3^{-3/4} \pi^{1/2} \left(e^{3/4}\sqrt{N} + \bar{e}^{3/4}\sqrt{\bar{N}}\right). \tag{2.7}$$

Equations (2.5) and (2.6) can be used in (2.7) to eliminate $\bar{N}$ and e in favor of N, and the optimal value of N determined by maximizing $s_{\rm FT}$ at fixed $Q_{\rm FT}$ and $m_{\rm FT}$. Requiring that $Q_{\rm FT} = Q_{\rm SG}$ and $m_{\rm FT} = m_{\rm SG}$, the resulting equilibrium values of N, $\bar{N}$ and e can then be expressed in terms of the supergravity parameters r_h and α:

$$N = \frac{\pi^{5/2}}{2\kappa} r_h^4 e^{2\alpha}\,, \qquad \bar{N} = \frac{\pi^{5/2}}{2\kappa} r_h^4 e^{-2\alpha}\,, \qquad e = \bar{e} = \frac{3\pi^3}{4\kappa^2} r_h^4\,. \tag{2.8}$$

Inserting these expressions into (2.7), we obtain agreement with the supergravity entropy (2.2), up to a numerical coefficient: $s_{\rm SG} = 2^{3/4} s_{\rm FT}$. Given the dependence (2.7) of the microscopic entropy on the gas energy, we see that, as noted in the Introduction, the supergravity entropy behaves as if *each* gas carried *twice* the available energy.

For later use, let us also recall that the energies of the two SYM gases are related to the corresponding temperatures through

$$e = \frac{3\pi^2}{8} N^2 T^4, \qquad \bar{e} = \frac{3\pi^2}{8} \bar{N}^2 \bar{T}^4\,, \tag{2.9}$$

so at equilibrium we have

$$T = \frac{2^{3/4}}{\pi r_h e^{\alpha}}, \qquad \bar{T} = \frac{2^{3/4}}{\pi r_h e^{-\alpha}}\,. \tag{2.10}$$

As noted in [1], the overall temperature of the system, $T_{\rm FT} \equiv (\partial s_{\rm FT}/\partial m_{\rm FT})^{-1}_{Q_{\rm FT}}$, can be expressed in terms of the gas temperatures through

$$\frac{2}{T_{\rm FT}} = \frac{1}{T} + \frac{1}{\bar{T}}\,,$$

and is therefore a factor of $2^{3/4}$ larger than the Hawking temperature (2.3), as expected from the numerical discrepancy between $s_{\rm FT}$ and $s_{\rm SG}$.

3. Microscopic absorption probabilities

In the model the stack of branes is decoupled from the stack of antibranes, so, at least for low enough frequencies, the probability that the system absorbs quanta of a given field with frequency ω must be given by the sum of the two independent contributions,

$$P^{(l)}_{\rm FT}(\omega) = P^{(l)}\left(\omega; N, T\right) + P^{(l)}\left(\omega; \bar{N}, \bar{T}\right)\,, \tag{3.1}$$

where $P^{(l)}(\omega; N, T)$ denotes the probability of absorption by a strongly-coupled SU(N) SYM gas with temperature T. In the next subsection we will determine this probability. For simplicity, we will consider only absorption of a minimal scalar field with low frequency, in the sense that

$$\omega \ll T, \bar{T}\,, \tag{3.2}$$

which through (2.10) is seen to imply $\omega \ll 1/r_h$.

3.1 Absorption by a strongly-coupled $\mathcal{N}=4$ SYM gas (or, equivalently, by a near-extremal threebrane)

Just as was done for the entropy calculation in [1], we will use the AdS/CFT correspondence [21] to map the SYM absorption calculation onto the problem of absorption by a *near-extremal* black threebrane. The latter's throat radius R and horizon radius r_0 are related to the gauge group rank N and gas temperature T through the well-known expressions

$$R^4 = 4\pi g_s N l_s^4 = \frac{\kappa N}{2\pi^{5/2}}\,, \qquad r_0 = \pi T R^2 = T\sqrt{\frac{\kappa N}{2\pi^{1/2}}}\,. \tag{3.3}$$

Of course, these formulas are simply a rewriting of the general expressions (2.4) and (2.3) in the near-extremal limit, $r_h \to 0$ with $r_h^2 e^{\alpha}$ fixed. *It is very important, however, not to confuse r_0 and R^4, which should be regarded as auxiliary parameters in the microscopic calculation, with r_h and $r_h^4 \sinh^2\alpha$, which are the parameters that characterize the arbitrarily far-from-extremal black brane whose absorption probabilities we will attempt to reproduce.*

The absorption probability for a minimal scalar field ϕ with frequency (3.2) on the background of a near-extremal black threebrane has been computed in [22].[5] There are a few errors and misprints in that calculation, however, that we will now correct.

The radial equation of motion for the lth partial wave is

$$\partial_\rho^2\phi + \frac{5\rho^4-\rho_0^4}{\rho(\rho^4-\rho_0^4)}\partial_\rho\phi - \frac{\rho^2 l(l+4)}{\rho^4-\rho_0^4}\phi + \frac{\rho^4(\rho^4+\mathcal{R}^4)}{(\rho^4-\rho_0^4)^2}\phi = 0\,, \tag{3.4}$$

where following [22] we have defined

$$\rho = \omega r\,, \qquad \rho_0 = \omega r_0\,, \qquad \mathcal{R} = \omega R\,. \tag{3.5}$$

The calculation in [22] assumes that

$$\rho_0 \ll \mathcal{R} \ll 1\,. \tag{3.6}$$

In the *outer region* $\rho \gg \rho_0$, (3.4) reduces to

$$\partial_\rho^2\phi + \frac{5}{\rho}\partial_\rho\phi + \left(1 + \frac{\mathcal{R}^4}{\rho^4} - \frac{l(l+4)}{\rho^2}\right)\phi = 0\,. \tag{3.7}$$

Notice that the ratio between the second and third terms inside the large parentheses is of order

$$\frac{\mathcal{R}^4}{\rho^2} = \left(\frac{\omega R^2}{r}\right)^2 \ll \left(\frac{\omega R^2}{r_0}\right)^2 = \left(\frac{\omega}{\pi T}\right)^2 \ll 1\,,$$

where we have made use of (3.3) and (3.2). So in the outer region, *independently* of whether or not $r \gg R$ ($\rho \gg \mathcal{R}$), the $\mathcal{R}^4$ term in (3.7) can be dropped.[6] The first term inside the large parentheses, on the other hand, will be relevant for $\rho \gg 1$, and so cannot be dropped. The resulting equation is related to Bessel's equation, and the general solution can be

[5] The same work also computes the probability in the opposite regime, $\omega \gg T$, correcting the previous results [23].

[6] In [22], the outer region is defined instead as $r \gg R$, but then it would not overlap with the inner region, which, as we will see below (and contrary to what is stated in [22]), must be restricted to $r \ll R$.

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written as

$$\phi(\rho) = \frac{A_l}{\rho^2} J_{l+2}(\rho) + \frac{B_l}{\rho^2} N_{l+2}(\rho)\,. \tag{3.8}$$

Using the asymptotic form of the Bessel functions for $\rho \gg 1$, the ingoing flux at infinity then follows as

$$\begin{aligned}\mathcal{F}^{(\mathrm{in})}_{r\to\infty} &\equiv \lim_{r\to\infty} \frac{f(r) r^5}{2i}\left(\phi^{(\mathrm{in})*}\partial_r\phi^{(\mathrm{in})} - \phi^{(\mathrm{in})}\partial_r\phi^{(\mathrm{in})*}\right) \\ &= -\frac{|A_l + iB_l|^2}{2\pi\omega^4}\,. \end{aligned}\tag{3.9}$$

On the other hand, for $\rho \ll 1$ (which includes in particular the region $\rho_0 \ll \rho \ll \mathcal{R}$), we have

$$\phi(\rho) = \frac{A_l \rho^l}{2^{l+2}(l+2)!} - \frac{B_l\, 2^{l+2}(l+1)!}{\pi \rho^{l+1}}\,. \tag{3.10}$$

Consider now the *inner region* $\rho_0 \leq \rho \ll \mathcal{R} \ll 1$. Defining $x = \rho_0^2/\rho^2$ as in [22], (3.4) can be rewritten as

$$\partial_x^2\phi - \frac{1+x^2}{x(1-x^2)}\partial_x\phi - \frac{l(l+4)}{4x^2(1-x^2)}\phi + \frac{\mathcal{R}^4/\rho_0^2 + \rho_0^2/x^2}{4x(1-x^2)^2}\phi = 0\,. \tag{3.11}$$

The ρ_0^2/x^2 term can be neglected in comparison with $\mathcal{R}^4/\rho_0^2$, since this just amounts to the statement that $\rho \ll \mathcal{R}$, which defines the inner region.[7] At the same time, by assumption we know that $\mathcal{R}^4/\rho_0^2 = (\omega/\pi T)^2 \ll 1$, and so the last term in (3.11) is seen to be completely irrelevant unless one is very close to the horizon, $x \simeq 1$, where it gives the dominant contribution and implies that $\phi(\rho) \propto (1-x^2)^{\pm i\mathcal{R}^4/4\rho_0^2}$. Choosing the lower sign in order for the solution to be purely ingoing at the horizon, one is thus led to the conclusion that

$$\phi(\rho) = (1-x^2)^{-i\mathcal{R}^2/4\rho_0}\varphi(x)\,, \tag{3.12}$$

where, to leading order in $\mathcal{R}^2/\rho_0$, $\varphi(x)$ satisfies equation (3.11) with the last term omitted, and is by construction regular at the horizon, $x = 1$.

Given (3.12), the ingoing flux at the horizon follows as

$$\begin{aligned}\mathcal{F}^{(\mathrm{in})}_{r\to r_0} &\equiv \lim_{r\to r_0} \frac{f(r) r^5}{2i}\left(\phi^*\partial_r\phi - \phi\partial_r\phi^*\right) \\ &= -\frac{\mathcal{R}^2\rho_0^3|\varphi(1)|^2}{\omega^4}\,. \end{aligned}\tag{3.13}$$

Combining (3.9) and (3.13) we can write down the desired absorption probability[8]

$$P^{(l)}(\omega; N, T) \equiv \frac{\mathcal{F}^{(\mathrm{in})}_{r\to r_0}}{\mathcal{F}^{(\mathrm{in})}_{r\to\infty}} = 2\pi\mathcal{R}^2\rho_0^3 \frac{|\varphi(1)|^2}{|A_l + iB_l|^2}\,. \tag{3.14}$$

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[7] As mentioned in the previous footnote, in [22] the inner region is defined simply as $\rho_0 \leq \rho \ll 1$, and it is incorrectly stated that throughout this region ρ_0^2/x^2 can be dropped compared to $\mathcal{R}^4/\rho_0^2$.

[8] The factor $|\varphi(1)|^2$ was erroneously omitted from the calculation in [22], so their absorption probabilities are off by this (l-dependent) constant.

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To complete the calculation, we need to determine $\varphi(1)$ and $\varphi(x \ll 1)$ (which compared against (3.10) will allow us to identify A_l and B_l). As explained in [22], the function $\varphi(x)$ can be expressed in terms of hypergeometric functions:

$$\varphi(x) = D_l \varphi_1(x) + C_l \varphi_2(x)\,, \tag{3.15}$$

$$\varphi_1(x) = x^{2+l/2} F\left(1+\frac{l}{4}, 1+\frac{l}{4}; 2+\frac{l}{2}; x^2\right), \tag{3.16}$$

$$\varphi_2(x) = x^{-l/2} F\left(-\frac{l}{4}, -\frac{l}{4}; -\frac{l}{2}; x^2\right). \tag{3.17}$$

The coefficients D_l, C_l must be determined by requiring φ to be smooth at $x = 1$, with help of the relation

$$\begin{aligned} F(a,b;c;z) = & \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} F(a,b;a+b-c+1;1-z) + \\ & + (1-z)^{c-a-b} \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)} F(c-a, c-b; c-a-b+1; 1-z)\,. \end{aligned}$$

Since in our case $c = a + b$, the above relation must be regularized by letting $c \to c + \epsilon$. Depending on the value of l, there are three different cases to consider [22]:

- **For odd values of** l, the $\log(1-x^2)$ singularities in $\varphi_1(x \to 1)$ and $\varphi_2(x \to 1)$ cancel if we choose

  $$D_l = 1, \qquad C_l = -\frac{\Gamma(2+l/2)\Gamma(-l/4)^2}{\Gamma(1+l/4)^2\Gamma(-l/2)}\,.$$

  One can then deduce that

  $$\varphi(1) = (-1)^{(l-1)/2} 2\pi \frac{\Gamma(2+l/2)}{\Gamma(1+l/4)^2}\,,$$

  whereas for $x \ll 1$ ($\rho \gg \rho_0$) we have

  $$\phi(x) \simeq \varphi(x) \simeq C_l (\rho/\rho_0)^l\,.$$

  Matching this with the outer region's solution (3.10), we see that to this order

  $$A_l \simeq 2^{l+2}(l+2)! C_l \rho_0^{-l}, \qquad B_l \simeq 0\,.$$

  Employing this and the value of $\phi(1)$ in the master formula (3.14), we finally conclude that

  $$P^{(l)}(\omega; N, T) = \frac{2^{-2l-1}\pi^3}{(l+2)!^2} \frac{\Gamma(-l/2)^2}{\Gamma(-l/4)^4} \omega^{2l+5} r_0^{2l+3} R^2\,, \tag{3.18}$$

  with $r_0(N,T)$ and $R(N)$ given by (3.3).

- **For** l **and** $l/2$ **even**, φ_1 still has a logarithmic singularity, but φ_2 becomes a Legendre polynomial,

  $$\varphi_2(x) = \frac{n!^2}{(2n)!} P_n(2/x^2 - 1)\,.$$

We can then set $D_l = 0$, $C_l = 1$, and deduce that

$$\varphi(1) = \frac{n!^2}{(2n)!}$$

and

$$\varphi(x \ll 1) \simeq (\rho/\rho_0)^l \quad \Longrightarrow \quad A_l \simeq 2^{l+2}(l+2)!\rho_0^{-l}, \quad B_l \simeq 0\,.$$

So in this case

$$P^{(l)}(\omega; N, T) = \frac{2^{-2l-3}\pi}{(l+2)!^2}\frac{(l/4)!^4}{(l/2)!^2}\omega^{2l+5}r_0^{2l+3}R^2\,. \tag{3.19}$$

- **For l even, $l/2$ odd**, φ_1 is still singular and φ_2 is ill-defined (and its regularized version is proportional to φ_1). The desired non-singular solution can be expressed in terms of Meijer's G function [24],

$$\varphi(x) = G^{20}_{22}\left(x^2 \,\middle|\, \begin{matrix} 1 & 1 \\ -l/4 & 1+l/4 \end{matrix}\right),$$

which implies that

$$\varphi(1) = 1$$

and[9]

$$\varphi(x \ll 1) \simeq \frac{(l/2)!}{\Gamma(1+l/4)^2}(\rho/\rho_0)^l \quad \Longrightarrow \quad A_l \simeq \frac{2^{l+2}(l+2)!(l/2)!}{\Gamma(1+l/4)^2}\rho_0^{-l}, \quad B_l \simeq 0\,.$$

It follows that

$$P^{(l)}(\omega; N, T) = \frac{2^{-2l-3}\pi}{(l+2)!^2}\frac{\Gamma(1+l/4)^4}{(l/2)!^2}\omega^{2l+5}r_0^{2l+3}R^2\,, \tag{3.20}$$

which is in fact the same formula as (3.19).

Using the identity $\Gamma(x) = \pi/[\sin(\pi x)\Gamma(1-x)]$, the result (3.18) for odd l can be put in the form

$$P^{(l)}(\omega; N, T) = \frac{2^{-2l-3}\pi}{(l+2)!^2}\frac{\Gamma(1+l/4)^4}{\Gamma(1+l/2)^2}\omega^{2l+5}r_0^{2l+3}R^2\,, \tag{3.21}$$

which agrees with (3.20) and is therefore seen to hold for all values of l.

3.2 Predictions of the brane-antibrane model

In the previous subsection we have seen that the absorption probability for an $\mathcal{N} = 4$ SU(N) SYM gas with temperature T takes the form (3.21), with R and r_0 the functions of N and T specified in (3.3). We will now use this result to determine the explicit form of the microscopic absorption probabilities (3.1).

As we reviewed in section 2, the model predicts that the numbers of branes and antibranes are given by (2.8), and the temperatures of the two gases are as indicated in (2.10). Combining these with (3.3), we see that the parameters to be used in (3.21) are

$$R^2 = \frac{1}{2}r_h^2 e^{\alpha}\,, \qquad r_0 = 2^{-1/4}r_h\,, \qquad \bar{R}^2 = \frac{1}{2}r_h^2 e^{-\alpha}\,, \qquad \bar{r}_0 = 2^{-1/4}r_h\,. \tag{3.22}$$

[9]The $(l/2)!$ factor is missing in [22].

Plugging this into (3.21) and then (3.1), we get our final prediction for the microscopic absorption probabilities,

$$P_{\rm FT}^{(l)}(\omega) = \frac{2^{-5l/2-15/4}\pi}{(l+2)!^2}\frac{\Gamma(1+l/4)^4}{\Gamma(1+l/2)^2}(\omega r_h)^{2l+5}\cosh\alpha\,. \tag{3.23}$$

4. Comparison with supergravity

4.1 Neutral case

Let us now compare the microscopic predictions (3.23) against the actual supergravity results, specializing first to the case of the neutral black threebrane, $\alpha = 0$, which is equivalent to the Schwarzschild black hole in seven spacetime dimensions. The corresponding absorption probabilities for arbitrary partial waves of a minimal scalar field have been computed in [25]:[10]

$$P_{\rm SG}^{(l)}(\omega) = \frac{2^{-3l-3}\pi^2}{(l+2)!^2}\frac{\Gamma(1+l/4)^2}{\Gamma(1/2+l/4)^2}(\omega r_h)^{2l+5}\,. \tag{4.1}$$

The functional dependence is in perfect agreement with (3.23) for $\alpha = 0$. Despite appearances, using the identity $\Gamma(x) = 2^{1-2x}\sqrt{\pi}\,\Gamma(2x)/\Gamma(x+1/2)$ the numerical coefficients can also be seen to agree, except for a power of two:

$$P_{\rm SG}^{(l)}(\omega) = 2^{3/4+l/2}P_{\rm FT}^{(l)}(\omega)\,. \tag{4.2}$$

Notice that this numerical discrepancy can be summarized in exactly the same manner as the one found for the entropy in [1]: if each gas could somehow carry twice the energy that is available to it, then according to (2.9) T and $\bar{T}$ would increase[11] by a factor of $2^{1/4}$, which as seen in (3.3) increases r_0 and $\bar{r}_0$ by the same factor, implying in turn, through (3.21) and (3.1), that $P_{\rm FT}^{(l)} \to 2^{3/4+l/2}P_{\rm FT}^{(l)}$. For $l = 0$ this is not a new result, for in fact the comparison in (4.2) is, for the special case of the s-wave, precisely the entropy comparison made in [1]: the absorption probabilities are of course proportional to the corresponding cross sections, and for the s-wave, the latter reduce at low frequencies to the respective horizon areas [26, 27], which are in turn, according to the Bekenstein-Hawking formula, proportional to the entropies.

4.2 Charged case

To the best of our knowledge, the absorption probability for the black threebrane arbitrarily far from extremality has not yet been computed, so we will need to work it out here, restricting again to the low-frequency regime (3.2). Fortunately, this will just amount to a simple generalization of the calculation in Section 3.1.

The radial equation of motion is again (3.4), now with the replacements $\rho_0 \to \rho_h$ and $\mathcal{R}^4 \to \rho_h^4 \sinh^2\alpha$, where $\rho_h \equiv \omega r_h$. In the *outer region* $\rho \gg \rho_h$ we still have the Bessel solution (3.8). The difference is that now, contrary to (3.6), we do not have a clear

[10] The $l = 0$ case had been worked out previously in [26, 27].

[11] It is perhaps worth pointing out that even in this case the Hawking and microscopic temperatures would not agree, but instead $T_{\rm FT} = 2T_H$.

separation between ρ_h and $\rho_h\sqrt{\sinh\alpha}$, and so we must define the *inner region* simply as $\rho_h \leq \rho \ll 1$, implying that both terms in the numerator of the last term of (3.11) are comparable. It is still true, however, that the last term of (3.11) is relevant only very close to the horizon ($x = 1$). We conclude then that, at the order we are interested in, the only change in the calculation is the modification of the exponent in (3.12) to

$$-\frac{i}{4}\sqrt{\frac{\rho_h^4\sinh^2\alpha}{\rho_h^2}+\rho_h^2} = -\frac{i}{4}\rho_h\cosh\alpha\,.$$

Carrying this change through in (3.13) and (3.14), and comparing with the result (4.1) for the neutral case, we deduce that for $\alpha \neq 0$ the absorption probability is modified into

$$P^{(l)}_{\rm SG}(\omega) = \frac{2^{-3l-3}\pi^2}{(l+2)!^2}\frac{\Gamma(1+l/4)^2}{\Gamma(1/2+l/4)^2}(\omega r_h)^{2l+5}\cosh\alpha\,, \tag{4.3}$$

exactly as predicted by the microscopic result (3.23) !

We conclude then that, for arbitrary charge,

$$P^{(l)}_{\rm SG}(\omega) = 2^{3/4+l/2}P^{(l)}_{\rm FT}(\omega)\,. \tag{4.4}$$

As explained in the previous subsection, this comparison was bound to work for $l = 0$, since in that case it is simply a rephrasing of the entropy comparison in [1]. The non-trivial results obtained in this paper are the infinite number of successful comparisons (4.4) for $l > 0$.

5. Hawking radiation

Given the absorption probabilities (4.3), the corresponding absorption cross-sections follow as [28]

$$\sigma^{(l)}_{\rm SG}(\omega) = \frac{8\pi^2}{3\omega^5}(l+1)(l+2)^2(l+3)P^{(l)}_{\rm SG}(\omega)\,. \tag{5.1}$$

These in turn allow us to compute the rates of Hawking radiation into each of the partial waves,

$$d\Gamma^{(l)}_{\rm SG}(\omega) = \frac{\sigma^{(l)}_{\rm SG}(\omega)\omega}{e^{\omega/T_H}-1}\,d\omega\,, \tag{5.2}$$

where $\sigma^{(l)}_{\rm SG}(\omega)$ plays the role of greybody factor.

From the microscopic perspective, given that the brane and antibrane subsystems are decoupled, we expect, in analogy to (3.1),

$$d\Gamma^{(l)}_{\rm FT}(\omega) = d\Gamma^{(l)}(\omega;N,T) + d\Gamma^{(l)}(\omega;\bar{N},\bar{T})\,,$$

where the rates on the right-hand side refer to bulk radiation emerging from the two $\mathcal{N}=4$ SYM gases. But again, through the AdS/CFT correspondence, these should be equivalent to the rates of emission for the corresponding *near-extremal* black threebranes, which are given by formulas analogous to (5.2). We thus have

$$d\Gamma^{(l)}_{\rm FT}(\omega) = \left[\frac{\sigma^{(l)}_{\rm FT}(\omega;N,T)}{e^{\omega/T}-1}+\frac{\sigma^{(l)}_{\rm FT}(\omega;\bar{N},\bar{T})}{e^{\omega/\bar{T}}-1}\right]\omega\,d\omega\,, \tag{5.3}$$

which does not resemble (5.2) in any obvious way.

In the low-frequency regime (3.2) where we are working, the supergravity and microscopic emission rates simplify to

$$d\Gamma^{(l)}_{\rm SG}(\omega) = \sigma^{(l)}_{\rm SG}(\omega) T_H \, d\omega \tag{5.4}$$

and

$$d\Gamma^{(l)}_{\rm FT}(\omega) = \left[\sigma^{(l)}_{\rm FT}(\omega; N, T) T + \sigma^{(l)}_{\rm FT}(\omega; N, T)\bar{T}\right] d\omega \,, \tag{5.5}$$

respectively. Using (5.1), we see that the comparison between these rates is equivalent to the comparison of

$$P^{(l)}_{\rm SG}(\omega) T_H \quad \text{vs.} \quad P^{(l)}(\omega; N, T) T + P^{(l)}(\omega; \bar{N}, \bar{T})\bar{T} \,,$$

which is clearly independent from the successful match between (3.1) and (4.3). Nevertheless, combining (4.3) and (2.3) we see that

$$P^{(l)}_{\rm SG}(\omega) T_H = \frac{2^{-3l-3}\pi}{(l+2)!^2} \frac{\Gamma(1+l/4)^2}{\Gamma(1/2+l/4)^2} \omega^{2l+5} r_h^{2l+4} \,,$$

whereas using (3.21) with (3.3), (2.8) and (2.10) we obtain

$$P^{(l)}(\omega; N, T) T + P^{(l)}(\omega; \bar{N}, \bar{T})\bar{T} = \frac{2^{-5l/2-3}}{(l+2)!^2} \frac{\Gamma(1+l/4)^4}{\Gamma(1+l/2)^2} \omega^{2l+5} r_h^{2l+4} \,,$$

so that we again have a perfect functional match!

Just like in the previous section, despite their superficial dissimilarity the numerical coefficients also agree, except for the power of two that corresponds to doubling the energy of each gas (which translates into increasing T and $\bar{T}$ by a factor of $2^{1/4}$, and setting $r_0 = \bar{r}_0 = r_h$, rather than $2^{-1/4} r_h$ as in (3.22)). In short, we have found that

$$\Gamma^{(l)}_{\rm SG}(\omega) = 2^{l/2} \Gamma^{(l)}_{\rm FT}(\omega) \,. \tag{5.6}$$

Equally important, we have learned that, at least to lowest order in the frequency, the separate Hawking radiation rates for the D3-brane and $\overline{\rm D3}$-brane stacks *agree* with one another,

$$d\Gamma^{(l)}(\omega; N, T) = d\Gamma^{(l)}(\omega; \bar{N}, \bar{T}) \,. \tag{5.7}$$

This is in spite of the fact that the two gases have *different* temperatures. As a matter of fact, using (3.3) in (3.21) one sees that the product $P^{(l)}(\omega; N, T) T$ which controls the D3-brane emission rate depends on N and T only through the combination $N^2 T^4$, which is of course the energy density (2.9) of the corresponding gas. We conclude then that the D3 and $\overline{\rm D3}$ radiation rates agree precisely because the two gases have *equal* energies! At the very least, this is an important self-consistency check for the model: since one postulates that the two gas energies are the same, it is satisfying to see that this equality will not be disturbed when the black brane radiates, which is part of what it does for a living. But one can actually view this as an *explanation* of the equal-energy condition: if the energies were initially different (for instance, on account of the gases having equal temperatures), then the gas with higher energy would radiate more, and the energies would tend to equalize. It is only the equal-energy case that is in this sense 'stable'.

On the other hand, we have seen in the previous section that the D3 and $\overline{\mathrm{D3}}$ absorption probabilities are in fact *different*, which implies that, when we disturb the black brane by throwing some radiation at it, the component with the lowest temperature (or, equivalently, the largest number of branes) will absorb more energy. This suggests that the supergravity counterpart of the microscopic system with unequal gas energies is some type of excited state of the black brane, which will eventually relax back the preferred equal-energy configuration. Notice that the corresponding solution will in general not possess the same symmetry properties as the original threebrane solution. For instance, after absorption of $l > 0$ radiation one would expect the black brane to become distorted into some configuration that is no longer spherically symmetric.

It would clearly be of great interest to establish whether the above findings continue to hold at next-to-leading (or perhaps even higher) order in the radiation frequency, or if they are just somehow a special property of the lowest-order terms. Leaving a detailed study of this question for future work, let us just remark at this point that at higher order the stability analysis would be more involved, for one would for instance have to take into account the possibility that part of the radiation emitted by one of the stacks is absorbed by the other.

6. Conclusions

6.1 Summary of results

We have demonstrated that the brane-antibrane model formulated in [1] can correctly account for the low-frequency absorption probabilities and Hawking emission rates of the black threebrane arbitrarily far from extremality, for arbitrary partial waves of a minimal scalar field. Our main results, the comparisons (4.4) and (5.6), amount to an infinite number of new tests of the microscopic model. Notice that these tests are indeed independent from one another. One might for instance suspect that since the passage from one partial wave to the next involves an additional derivative in the brane-bulk coupling, it is bound to give rise to a factor of ω^2, and by dimensional analysis, r_h^2, in the absorption probability. In this way all of the absorption results for higher partial waves would be related to the $l = 0$ case, which, as explained in Section 4.1, is in fact nothing but the entropy comparison in [1]. That this is in general not the whole story can be seen by noting that the l-dependence of the exponent of the frequency is *not* the same in, for instance, the extremal ($P \propto \omega^{4l+8}$) [29, 30] and near-extremal ($P \propto \omega^{2l+5}$) [22] cases. And in any event, dimensional analysis obviously does not control the comparison between the numerical coefficients in the microscopic and supergravity sides, which has been seen to be successful for all partial waves, up to the same factor of 2 in the gas energies.

It is interesting that, as can be seen by combining (2.9) and (3.3), the equal-energy condition amounts to the statement that the horizon radii of the two near-extremal branes employed in the microscopic side coincide, $r_0 = \bar{r}_0$, and then the rescaling needed to resolve the numerical discrepancy identifies these with the horizon radius of the brane in the supergravity side, $r_0 = \bar{r}_0 = r_h$, as noted for instance above (5.6). This is essentially the agreement [1, 4, 5] mentioned in the Introduction between r_h and the transverse size of the microscopic system, $\sqrt{\langle\Phi^2\rangle}$, except that here we *are* keeping track the numerical coefficient.

One possible source of confusion is the fact that, as in previous analyses of the model, we have employed supergravity for the calculations in the *microscopic* side (see Sections 3.1 and 5). More concretely, what could seem suspicious in this paper is that the microscopic and supergravity absorption probabilities are obtained by solving the *same* equation of motion, namely, (3.4). One should not however lose sight of the fact that, as was emphasized in [1, 3], the results to be compared are extracted from two completely different regimes (near-extremal vs. arbitrarily-far-from-extremal) of (3.4). Moreover, in the microscopic side the parameters r_0 and R are not chosen in an *ad hoc* manner, but fixed by a maximization procedure. (The only condition that *is* imposed by hand is the equality of the energies of the two gases.) And, perhaps most significant of all, we do not simply compare one supergravity absorption probability against another, but one against the *sum* of two others, in a setup where the three corresponding brane charges are in general all *different* from one another.

To bring out more clearly the precise sense in which the agreement found in the absorption calculation is non-trivial, imagine we had access to the *exact* absorption probability that follows from the equation of motion (3.4), which we could denote as $P_{(l)}(\omega; r_0, (R/r_0)^2)$. Then the supergravity probability is of course

$$P^{(l)}_{\mathrm{SG}}(\omega) = P_{(l)}(\omega; r_h, \sinh\alpha)\,.$$

The prescription of the model, on the other hand, proceeds in three steps. We first consider the near-extremal limit of the full absorption probability, $P^{\mathrm{NE}}_{(l)}(\omega; r_0, (R/r_0)^2)$, which by definition is *just the leading term* of $P_{(l)}(\omega; r_0, (R/r_0)^2)$ for arbitrarily large (but finite) $(R/r_0)^2$. Second, we apply this formula separately to the brane and antibrane subsystems with the parameters predicted by the model, namely, (3.22). To avoid the numerical discrepancy, for the purpose of this discussion we will from the start set $r_0 = \bar{r}_0 = r_h$, rather than $2^{-1/4}r_h$. Third, the microscopic absorption probability is obtained from the sum of the brane and antibrane contributions, i.e.,

$$P^{(l)}_{\mathrm{FT}}(\omega) = P^{\mathrm{NE}}_{(l)}\left(\omega; r_h, \frac{e^{\alpha}}{2}\right) + P^{\mathrm{NE}}_{(l)}\left(\omega; r_h, \frac{e^{-\alpha}}{2}\right).$$

For $P_{(l)}(\omega; r_h, \sinh\alpha)$ an arbitrary function, there is clearly no reason whatsoever for $P^{(l)}_{\mathrm{FT}}(\omega)$ to agree with $P^{(l)}_{\mathrm{SG}}(\omega)$. Nevertheless, we have found that, to lowest order in the frequency, $P_{(l)}(\omega; r_0, (R/r_0)^2)$ takes the form $p(\omega r_0)\sqrt{1+(R/r_0)^4}$, which is precisely as required to pass this test. It is far from obvious whether this pattern can continue to hold at higher order.

As we have emphasized in section 5, the comparison of the Hawking radiation rates brings in an entirely new requirement. From a physical perspective, the agreement (5.6) between the microscopic and supergravity emission rates is perhaps even more striking than that between the absorption probabilities, in particular because it is completely independent from the entropy match found in [1]. Notice, however, that at least at this order the black brane does not strictly speaking 'know' that it is made of two independent components that emit radiation independently, because the two different temperatures are not *directly* visible in the supergravity side. This is unlike the situation in, e.g., the D1-D5 system, where inclusion of the greybody factor is explicitly seen to convert the single thermal factor for emission at the Hawking temperature into the product (rather than the sum, as we have in our case) of two thermal factors corresponding to different temperatures [31]. The difference between the two cases is of course due to the emission mechanism: while in [31] the emission of a massless closed string into the bulk necessarily involves an interaction between the two gases, in our case the two gases are free to radiate independently. A higher-order calculation might conceivably allow one to see the two independent temperatures somewhat more explicitly in the supergravity side, but of course it is far from clear whether the agreement between the microscopic and supergravity emission rates can persist in such a calculation.

From the Hawking radiation analysis we have also learned that (at least for low frequencies) the rate of emission for each of the two components of the microscopic system depends only on the energy of the corresponding gas. If the gas energies were not equal, then the gas with higher energy would radiate more, and so the energies would tend to equalize. Our results can therefore be viewed as an explanation for the equal-energy condition, which up to now has been simply a postulate of the model. As we have also discussed in Section 5, however, the fact that the D3-brane and $\overline{\text{D3}}$-brane absorption probabilities are different implies that it should be possible to achieve unequal gas energies when we disturb the system by throwing radiation at it. This in turn suggests that the supergravity counterpart of the microscopic configuration with different gas energies should be some type of excited state of the black brane.

As has been noted already at various points in the above discussion, it is an important outstanding problem to establish whether the absorption and emission probabilities continue to agree at higher order in the radiation frequency. We hope to report on this question in future work.

To summarize, in this paper we have found significant new evidence in favor of the brane-antibrane model, and have moreover thrown light on the equal-energy condition which was heretofore one of its most puzzling aspects.

6.2 Critical assessment of the model

In spite of the successes of the model (which include in particular the results reported in this paper), the situation is not yet entirely satisfactory, and more work will be needed to conclusively validate the model. In preparation for it, it seems useful to include here a list of the various assumptions of the model that could be in need of more careful scrutiny:

1. The model assumes that the D3-$\overline{\text{D3}}$ pairs do not annihilate even partially, or in other words, that the open string vacuum is, at the relevant temperatures, stable. A calculation supporting this assumption was included in [1], and additional evidence

has been provided by the numerical analysis in [4]. As mentioned in the Introduction, the interpretation of [6, 8] can perhaps be viewed as a relaxation of this assumption.

2. It is assumed that all 3-$\overline{3}$ open strings acquire large masses and consequently cannot be excited. In other words, the model incorporates gases with $\mathcal{O}(N^2)$ and $\mathcal{O}(\bar{N}^2)$ degrees of freedom, but not $\mathcal{O}(N\bar{N})$. The lowest 3-$\overline{3}$ mode is of course the tachyon, which must by definition acquire a large positive mass-squared if the brane-antibrane system is to be stable. One must also consider the 3-$\overline{3}$ fermions (arising from the Ramond sector) that would at zero temperature be massless. The analysis of [1] provided some evidence that both of these modes indeed become massive enough to decouple.

3. The model does not include any binding energies. As emphasized in [4], at least naively one would in fact expect this to be wrong. In particular, it seems difficult to see how (e.g., gravitational) D3-$\overline{\text{D3}}$ binding energy could be avoided. This would be expected to arise from closed string exchange between the two stacks, or equivalently, from loops of 3-$\overline{3}$ open strings (which could contribute through virtual effects even if on-shell they are postulated to have large masses). If present, this type of binding energy would bring in some $N\bar{N}$-dependence.

4. The model involves a restriction on the types of brane-antibrane pairs that contribute: even arbitrarily far from extremality it is assumed that there is no contribution from D1-$\overline{\text{D1}}$ pairs. To understand this, it seems natural to try to extend the model to the case of the black brane which has *both* five-form and three-form RR charge [32]. A preliminary analysis appears to indicate that indeed D1-$\overline{\text{D1}}$ pairs disappear altogether when the three-form charge is taken to approach zero [33]. Other potentially relevant observations may be found in [34].

5. The model assumes that the component gases have equal energies, and consequently different temperatures. It is perhaps worth pointing out that one *can* in fact reproduce the supergravity entropy with a model based on equal-temperature gases, as long as one is willing to pay the price of fixing the number of branes and antibranes by hand, instead of choosing the value of N that maximizes the entropy. In this new scenario one would also lose many of the other successful predictions of the equal-energy model. It is therefore satisfying that the results of the present paper appear to provide for the first time a rationale for the equal-energy condition, in terms of 'stability' of the system with respect to Hawking emission.

6. The model assumes a specific form for the dynamics of the relevant gases: the formulas employed are those of strongly-coupled $\mathcal{N}=4$ SYM (or equivalently, a near-extremal black threebrane). Since the relevant temperatures are much lower than the string scale one would naively expect α' corrections to be suppressed. But in fact, when one writes down, e.g., the Born-Infeld action, there are some additional factors of N hidden in the non-abelian nature of the field strength, which can be seen to imply that the higher-derivative corrections are controlled not by l_s but by the scale $R =$

$(4\pi g_s N)^{1/4} l_s$, which is of course the throat radius of the corresponding supergravity solution [29, 35, 36]. What is very peculiar about the model is that, as can be seen by comparing (2.10) against (3.22), the predicted gas temperatures are precisely of order $1/R$, and so *a priori* one would have expected the higher-derivative corrections to play an important role,[12] in which case we would not be entitled to employ the SYM/near-extremal formulas.

It is clear from this list that the status of the brane-antibrane model is still open to debate. At the same time, the list certainly makes the body of evidence that has by now accumulated in favor of the model seem all the more remarkable.

The two most questionable assumptions are items 3 and 6. Notice that these two points are in fact not unrelated: the higher-derivative corrections of item 6 arise from massive open string modes, whose cummulative effect in loops is equivalent to closed string exchange, and is therefore associated with binding energy, this time of D3-D3 or $\overline{\text{D3}}$-$\overline{\text{D3}}$ type. A remarkable fact is that, if one attempts to incorporate the effect of these expected higher-derivative corrections into the energy and entropy formulas by dimensional analysis,[13] then *after* the maximization procedure one concludes that they modify the final entropy-mass relation only through a numerical factor![14] This might very well be, then, the origin of the ubiquitous factor of 2.

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References

[1] U.H. Danielsson, A. Güijosa and M. Kruczenski, *Brane-antibrane systems at finite temperature and the entropy of black branes*, *J. High Energy Phys.* **09** (2001) 011 [hep-th/0106201].

[2] U.H. Danielsson, A. Güijosa and M. Kruczenski, *Black brane entropy from brane-antibrane systems*, *Rev. Mex. Fis.* **49S2** (2003) 61–66 [gr-qc/0204010].

[12] It has been argued in [37, 38] (see also [39, 40, 41]) that at strong-coupling only the R^4F^4 correction is present. However, that argument relies heavily on supersymmetry, which is of course broken in our finite temperature setup. The effect of the temperature has been studied in [41].

[13] One might again have a chance of extracting the correct formulas for the strongly-coupled gas from supergravity, by invoking the generalization of AdS/CFT to higher energies, as in [29] and [35]–[42]. Notice, however, that one should *not* simply use the formulas for the (non-near-extremal) black threebrane, since according to the model (and physical intuition) that would include not only the desired contribution from the gas, but also from additional D3-$\overline{\text{D3}}$ pairs (which is precisely why one gets, for instance, negative specific heat).

[14] We thank Martín Kruczenski for emphasizing this point to us.

[3] A. Guijosa, H.H. Hernández Hernández and H.A. Morales Tecotl, *The entropy of the rotating charged black threebrane from a brane-antibrane system*, *J. High Energy Phys.* **03** (2004) 069 [hep-th/0402158].

[4] O. Saremi and A.W. Peet, *Brane-antibrane systems and the thermal life of neutral black holes*, *Phys. Rev.* **D 70** (2004) 026008 [hep-th/0403170].

[5] O. Bergman and G. Lifschytz, *Schwarzschild black branes from unstable D-branes*, *J. High Energy Phys.* **04** (2004) 060 [hep-th/0403189].

[6] S. Kalyana Rama, *A description of schwarzschild black holes in terms of intersecting M-branes and antibranes*, hep-th/0404026.

[7] G. Lifschytz, *Charged black holes from near extremal black holes*, *J. High Energy Phys.* **09** (2004) 009 [hep-th/0405042].

[8] S.K. Rama and S. Siwach, *A description of multicharged black holes in terms of branes and antibranes*, *Phys. Lett.* **B 596** (2004) 221 [hep-th/0405084].

[9] E. Halyo, *Black hole entropy and superconformal field theories on brane-antibrane systems*, hep-th/0406082.

[10] G.T. Horowitz, J.M. Maldacena and A. Strominger, *Nonextremal black hole microstates and U-duality*, *Phys. Lett.* **B 383** (1996) 151 [hep-th/9603109].

[11] M. Cvetič and D. Youm, *Entropy of non-extreme charged rotating black holes in string theory*, *Phys. Rev.* **D 54** (1996) 2612 [hep-th/9603147];
G.T. Horowitz, D.A. Lowe and J.M. Maldacena, *Statistical entropy of nonextremal four-dimensional black holes and U-duality*, *Phys. Rev. Lett.* **77** (1996) 430 [hep-th/9603195];
R. Kallosh and A. Rajaraman, *Brane-anti-brane democracy*, *Phys. Rev.* **D 54** (1996) 6381 [hep-th/9604193];
J.G. Zhou, H.J.W. Muller-Kirsten, J.Q. Liang and F. Zimmerschied, *M-branes, anti-M-branes and non-extremal black holes*, *Nucl. Phys.* **B 487** (1997) 155 [hep-th/9611146];
M.S. Costa and M.J. Perry, *Landau degeneracy and black hole entropy*, *Nucl. Phys.* **B 520** (1998) 205 [hep-th/9712026].

[12] D. Kabat, G. Lifschytz and D.A. Lowe, *Black hole thermodynamics from calculations in strongly coupled gauge theory*, *Phys. Rev. Lett.* **86** (2001) 1426 [hep-th/0007051];
D. Kabat, G. Lifschytz and D.A. Lowe, *Black hole entropy from non-perturbative gauge theory*, *Phys. Rev.* **D 64** (2001) 124015 [hep-th/0105171];
N. Iizuka, D. Kabat, G. Lifschytz and D.A. Lowe, *Probing black holes in non-perturbative gauge theory*, *Phys. Rev.* **D 65** (2002) 024012 [hep-th/0108006].

[13] O. Lunin and S.D. Mathur, *Statistical interpretation of Bekenstein entropy for systems with a stretched horizon*, *Phys. Rev. Lett.* **88** (2002) 211303 [hep-th/0202072];
S.D. Mathur, A. Saxena and Y.K. Srivastava, *Constructing 'hair' for the three charge hole*, *Nucl. Phys.* **B 680** (2004) 415 [hep-th/0311092];
S.D. Mathur, *Where are the states of a black hole?*, hep-th/0401115.

[14] S.S. Gubser and I. Mitra, *Instability of charged black holes in anti-de sitter space*, hep-th/0009126;
S.S. Gubser and I. Mitra, *The evolution of unstable black holes in anti-de Sitter space*, *J. High Energy Phys.* **08** (2001) 018 [hep-th/0011127];
H.S. Reall, *Classical and thermodynamic stability of black branes*, *Phys. Rev.* **D 64** (2001) 044005 [hep-th/0104071];

J.P. Gregory and S.F. Ross, *Stability and the negative mode for schwarzschild in a finite cavity*, *Phys. Rev.* **D 64** (2001) 124006 [hep-th/0106220].

[15] R. Gregory and R. Laflamme, *Black strings and p-branes are unstable*, *Phys. Rev. Lett.* **70** (1993) 2837 [hep-th/9301052]; *The instability of charged black strings and p-branes*, *Nucl. Phys.* **B 428** (1994) 399 [hep-th/9404071].

[16] G. Lifschytz, *Black hole thermalization rate from brane anti-brane model*, *J. High Energy Phys.* **08** (2004) 059 [hep-th/0406203].

[17] N. Iizuka, D. Kabat, G. Lifschytz and D.A. Lowe, *Quasiparticle picture of black holes and the entropy-area relation*, *Phys. Rev.* **D 67** (2003) 124001 [hep-th/0212246];
Stretched horizons, quasiparticles and quasinormal modes, *Phys. Rev.* **D 68** (2003) 084021 [hep-th/0306209].

[18] E. Halyo, *Gravitational entropy and string bits on the stretched horizon*, hep-th/0308166.

[19] S.S. Gubser, I.R. Klebanov and A.W. Peet, *Entropy and temperature of black 3-branes*, *Phys. Rev.* **D 54** (1996) 3915 [hep-th/9602135].

[20] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109];
S.S. Gubser, I.R. Klebanov and A.A. Tseytlin, *Coupling constant dependence in the thermodynamics of $N = 4$ supersymmetric Yang-Mills theory*, *Nucl. Phys.* **B 534** (1998) 202 [hep-th/9805156].

[21] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[22] G. Policastro and A. Starinets, *On the absorption by near-extremal black branes*, *Nucl. Phys.* **B 610** (2001) 117 [hep-th/0104065].

[23] Y. Satoh, *Propagation of scalars in non-extremal black hole and black p-brane geometries*, *Phys. Rev.* **D 58** (1998) 044004 [hep-th/9801125];
S. Musiri and G. Siopsis, *Temperature of D3-branes off extremality*, *Phys. Lett.* **B 504** (2001) 314 [hep-th/0003284];
J.F. Vázquez-Poritz, *Absorption by nonextremal D3-branes*, hep-th/0007202.

[24] I.S. Gradshteyn and I.M. Ryzhik, *Tables of integrals, series, and products*, Academic Press, 1994.

[25] P. Kanti and J. March-Russell, *Calculable corrections to brane black hole decay, I. The scalar case*, *Phys. Rev.* **D 66** (2002) 024023 [hep-ph/0203223].

[26] S.R. Das, G.W. Gibbons and S.D. Mathur, *Universality of low energy absorption cross sections for black holes*, *Phys. Rev. Lett.* **78** (1997) 417 [hep-th/9609052].

[27] R. Emparán, *Absorption of scalars by extended objects*, *Nucl. Phys.* **B 516** (1998) 297 [hep-th/9706204].

[28] S.S. Gubser, *Can the effective string see higher partial waves?*, *Phys. Rev.* **D 56** (1997) 4984 [hep-th/9704195].

[29] I.R. Klebanov, *World-volume approach to absorption by non-dilatonic branes*, *Nucl. Phys.* **B 496** (1997) 231 [hep-th/9702076].

[30] S.S. Gubser, I.R. Klebanov and A.A. Tseytlin, *String theory and classical absorption by three-branes*, *Nucl. Phys.* **B 499** (1997) 217 [hep-th/9703040];
I.R. Klebanov, I.W. Taylor and M. Van Raamsdonk, *Absorption of dilaton partial waves by D3-branes*, *Nucl. Phys.* **B 560** (1999) 207 [hep-th/9905174].

[31] J.M. Maldacena and A. Strominger, *Black hole greybody factors and D-brane spectroscopy*, *Phys. Rev.* **D 55** (1997) 861 [hep-th/9609026].

[32] J.G. Russo and A.A. Tseytlin, *Waves, boosted branes and BPS states in M-theory*, *Nucl. Phys.* **B 490** (1997) 121 [hep-th/9611047];
J.X. Lu and S. Roy, *((F, D1), D3) bound state and its t dual daughters*, *J. High Energy Phys.* **01** (2000) 034 [hep-th/9905014].

[33] A. Güijosa, unpublished.

[34] J.M. Maldacena and J.G. Russo, *Large-N limit of non-commutative gauge theories*, *J. High Energy Phys.* **09** (1999) 025 [hep-th/9908134];
R.G. Cai and N. Ohta, *On the thermodynamics of large-N non-commutative super Yang-Mills theory*, *Phys. Rev.* **D 61** (2000) 124012 [hep-th/9910092]; *Noncommutative and ordinary super Yang-Mills on $(D(p-2),Dp)$ bound states*, *J. High Energy Phys.* **03** (2000) 009 [hep-th/0001213];
D. Youm, *A note on (D(p-2),Dp) bound state and noncommutative Yang-Mills theory*, *Mod. Phys. Lett.* **A 15** (2000) 1949 [hep-th/0006019].

[35] S.S. Gubser, A. Hashimoto, I.R. Klebanov and M. Krasnitz, *Scalar absorption and the breaking of the world volume conformal invariance*, *Nucl. Phys.* **B 526** (1998) 393 [hep-th/9803023].

[36] S.P. de Alwis, *Supergravity, the DBI action and black hole physics*, *Phys. Lett.* **B 435** (1998) 31 [hep-th/9804019].

[37] S.S. Gubser and A. Hashimoto, *Exact absorption probabilities for the D3-brane*, *Commun. Math. Phys.* **203** (1999) 325 [hep-th/9805140].

[38] K.A. Intriligator, *Maximally supersymmetric RG flows and AdS duality*, *Nucl. Phys.* **B 580** (2000) 99 [hep-th/9909082].

[39] N.R. Constable and R.C. Myers, *Exotic scalar states in the AdS/CFT correspondence*, *J. High Energy Phys.* **11** (1999) 020 [hep-th/9905081];
M.S. Costa, *Absorption by double-centered D3-branes and the Coulomb branch of $N = 4$ sym theory*, *J. High Energy Phys.* **05** (2000) 041 [hep-th/9912073];
A test of the AdS/CFT duality on the coulomb branch, *Phys. Lett.* **B 482** (2000) 287 [hep-th/0003289];
L. Rastelli and M. Van Raamsdonk, *A note on dilaton absorption and near-infrared d3 brane holography*, *J. High Energy Phys.* **12** (2000) 005 [hep-th/0011044].

[40] U.H. Danielsson, A. Güijosa, M. Kruczenski and B. Sundborg, *D3-brane holography*, *J. High Energy Phys.* **05** (2000) 028 [hep-th/0004187].

[41] N.J. Evans, C.V. Johnson and M. Petrini, *Clearing the throat: irrelevant operators and finite temperature in large-N gauge theory*, *J. High Energy Phys.* **05** (2002) 002 [hep-th/0112058].

[42] X. Amador, E. Cáceres, H. García-Compean and A. Güijosa, *Conifold holography*, *J. High Energy Phys.* **06** (2003) 049 [hep-th/0305257].